

# Diffusion and entropy production for multi-networks with fitness factors

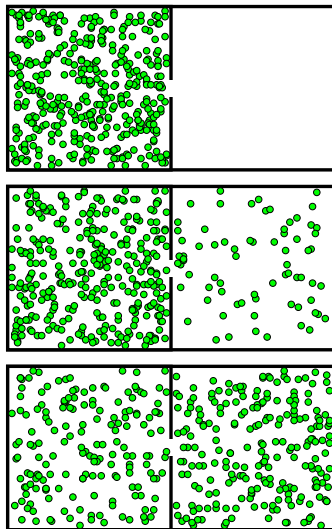
ECCS'13 WarmUp FlashTalk Session

Grzegorz Siudem and Janusz A. Hołyst

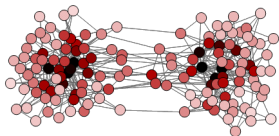
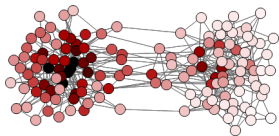
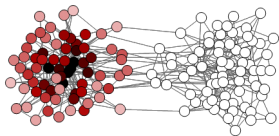
Faculty of Physics, Center of Excellence for Complex Systems Research  
Warsaw University of Technology

Barcelona, 14th September 2013

# Diffusion

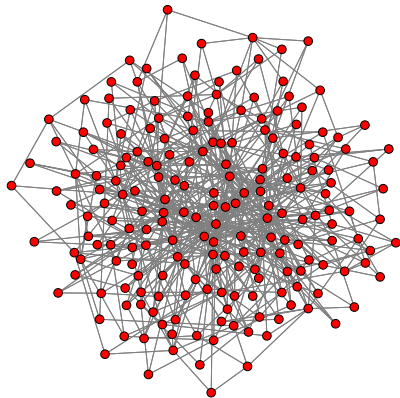


# Diffusion on networks

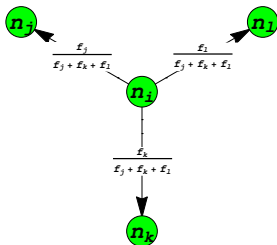


# Networks - definitions

- $A_{ij}$  - adjacency matrix of network,
- $n_i(t)$  - number of particles,
- $\sum_{i=1}^M n_i(t) = N$ ,
- $f_i$  - fitness factor,



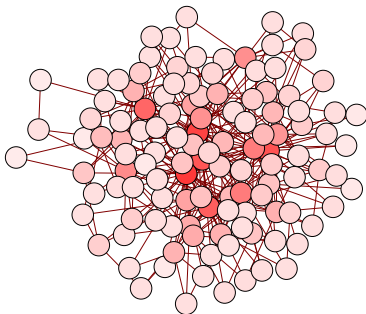
# Diffusion on networks - Markov chain approach



$$n_i(t+1) = f_i \sum_{k=1}^M \frac{A_{ik}}{g_k} n_k(t), \quad g_k = \sum_l A_{lk} f_l, \quad \mu_i = \frac{f_i g_i}{\sum_{k=1}^M f_k g_k},$$

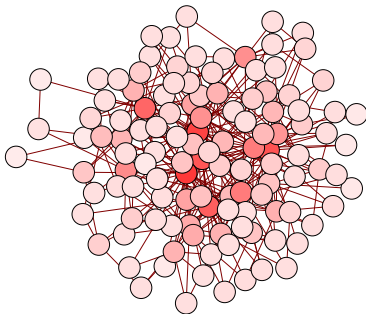
# Equilibrium (ergodic) distribution

$$\mu_i = \frac{f_i g_i}{\sum_{k=1}^M f_k g_k}$$

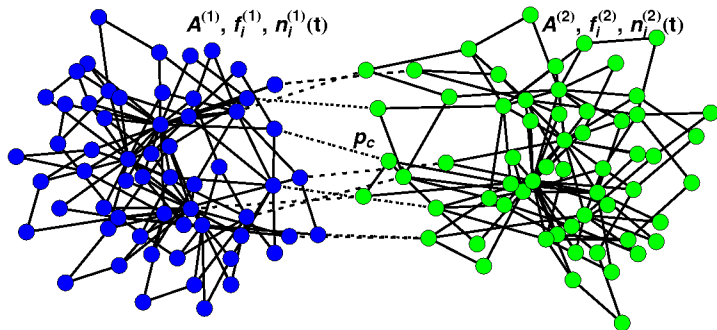


# Entropy of equilibrium distribution

$$S_{\infty} = -\frac{(f_i g_i \log(f_i g_i))_{\text{av}}}{(f_i g_i)_{\text{av}}} + \log(f_i g_i)_{\text{av}} + \log M.$$



# Coupled networks

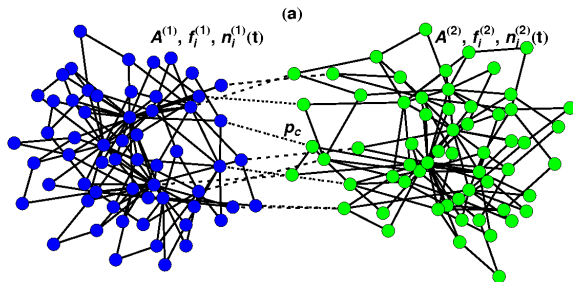




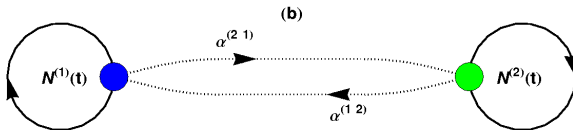
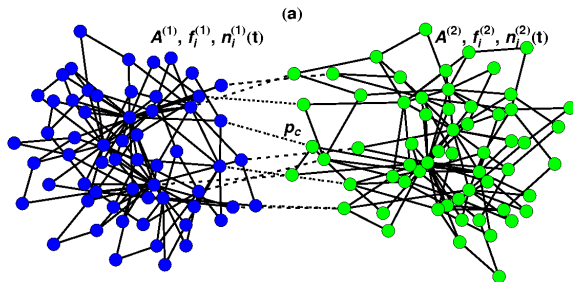
# Why diffusion on coupled complex networks?

- Social and technical networks have this structure.
- Classical diffusion between two crystals.
- Many more...

# Weak connection

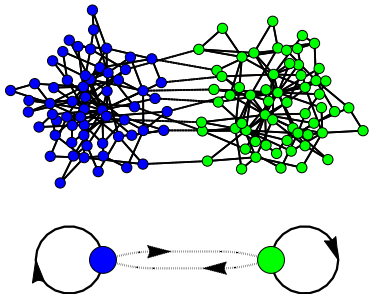


# Weak connection



# Separation of time scales

$$n_i^{(a)}(t) = N^{(a)}(t)\mu_i^{(a)}.$$

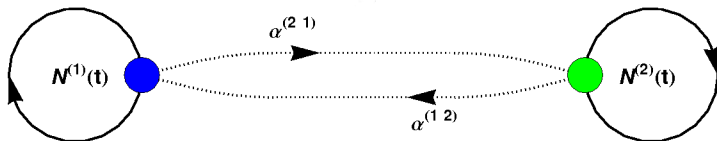


## Result of our approach

$$N^{(1)}(t+1) = (1 - \alpha^{(21)})N^{(1)}(t) + \alpha^{(12)}N^{(2)}(t),$$

where  $\alpha^{(ab)}$  are constants dependent on  $A^{(1)}$ ,  $A^{(2)}$ ,  $p_c$ ,  $\mathbf{f}^{(1)}$ ,  $\mathbf{f}^{(2)}$  as follows

$$\alpha^{(ba)} = p_c M^{(b)} \frac{f_{av}^{(a)} f_{av}^{(b)}}{(f_i^{(a)} g_i^{(a)})_{av}}$$



# First Fick's Law

For the diffusion on networks

$$\dot{N}^{(1)}(t) = - \left( N^{(1)}(t) - N^{(2)}(t) \right) D + FN,$$

where

$$D = \frac{\alpha^{(12)} + \alpha^{(21)}}{2}, \quad F = \frac{\alpha^{(21)} - \alpha^{(12)}}{2}.$$

# First Fick's Law

For the diffusion on networks

$$\dot{N}^{(1)}(t) = - \left( N^{(1)}(t) - N^{(2)}(t) \right) D + FN,$$

where

$$D = \frac{\alpha^{(12)} + \alpha^{(21)}}{2}, \quad F = \frac{\alpha^{(21)} - \alpha^{(12)}}{2}.$$

For classical diffusion

$$J = -D \frac{\partial \phi}{\partial x}$$

# Entropy and entropy production

$$S_{\text{total}}(t) = \left[ N^{(1)}(t) S_{\infty}^{(1)} + N^{(2)}(t) S_{\infty}^{(2)} \right] + \left[ -N^{(1)}(t) \log N^{(1)}(t) - N^{(2)}(t) \log N^{(2)}(t) + N \log N \right],$$

$$\sigma(t) = \dot{N}^{(1)}(t) \left( \log N^{(2)}(t) - \log N^{(1)}(t) + S_{\infty}^{(1)} - S_{\infty}^{(2)} \right),$$

where  $S_{\infty}^{(a)}$ ,  $a = 1, 2$ , are equilibrium entropies per particle.



# Hierarchical networks

$$\hat{f}_a = M^{(a)} f_{\text{av}}^{(a)}, \quad \hat{A}_{ab} = p_c^{(ab)}, \quad \text{for } a \neq b,$$

$$\hat{A}_{aa} = \frac{(f_i^{(a)} g_i^{(a)})_{\text{av}}}{M^{(a)} (f_{\text{av}}^{(a)})^2} \left( 1 - \sum_{r \neq a} \alpha^{(ra)} \right).$$

# Hierarchical networks

$$\hat{f}_a = M^{(a)} f_{\text{av}}^{(a)}, \quad \hat{A}_{ab} = p_c^{(ab)}, \quad \text{for } a \neq b,$$

$$\hat{A}_{aa} = \frac{(f_i^{(a)} g_i^{(a)})_{\text{av}}}{M^{(a)} (f_{\text{av}}^{(a)})^2} \left( 1 - \sum_{r \neq a} \alpha^{(ra)} \right).$$

I heard you like networks...

So we put a network into a network, so you can diffuse while you diffuse!



# Conclusions

With our approach...

- we found Fick's Law analogous for the diffusion on weakly coupled networks,
- we calculate diffusion constant and external force,

For those who prefer to read rather than to listen 😊

G. Siudem, J. A. Hołyst, *Diffusion and entropy production for multi-networks with fitness factors*, arXiv:1303.2650 [nlin.CD], (2013).

# Conclusions

With our approach...

- we can do calculation for the NONs,
- we propose tool for investigation diffusion on the hierarchical network, with different diffusion constant on every level of hierarchy.

For those who prefer to read rather than to listen 😊

G. Siudem, J. A. Hołyst, *Diffusion and entropy production for multi-networks with fitness factors*, arXiv:1303.2650 [nlin.CD], (2013).